MARKING GUIDE KAMSA MOCK 2022.

PURE MATHEMATICS. P 425/1

1. $4\cos y = 3\tan y + 3\sec y \qquad 0° \leq y \leq 360°$

$4\cos y = 3\frac{\sin y}{\cos y} + \frac{3}{\cos y}$ m_1

$4\cos^2 y = 3\sin y + 3$

$4(1-\sin^2 y) = 3\sin y + 3$

$4\sin^2 y + 3\sin y - 1 = 0$

let $\sin y = m$.

$4m^2 + 3m - 1 = 0$

$m = \frac{3 \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(8)}$ m_1

$m = -1 \quad , \quad m = 0.25$ m_1 (for m_1 and m_2)

$\sin y = -1$

$y = 90°$

$y = 270°$

m_1 (for both ms)

$\sin y = 0.25$

$y = 14.48°, 165.52°$

05

$\therefore y = \{14.48°, 165.52°, 270°$ A_1

2. $\int_0^{\pi/2} x \sin 2x \, dx$.

let $u = x$, $\frac{du}{dx} = 1$

$\frac{dv}{dx} = \sin 2x$ m_1

$v = -\frac{1}{2}\cos 2x$.

$\int_0^{\pi/2} x \sin 2x \, dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos 2x \, dx$ m_1

$= \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} + \frac{1}{2}\left[\frac{1}{2}\sin 2x\right]_0^{\pi/2}$ m_1

$= -\frac{1}{2}\left(\frac{\pi}{2}\right)\cos\pi - 0 + \frac{1}{2}\left(\frac{1}{2}\sin\pi - \frac{1}{2}\sin 0\right)$ m_1

$= \frac{\pi}{4}$ A_1

05

3. $5^{2t} = 5^{t+1} - 6.$

$(5^t)^2 - 5^t \cdot 5^1 + 6 = 0$

let $5^t = m.$

$m^2 - 5m + 6 = 0$ m1

$m = \dfrac{5 \pm \sqrt{(-5)^2 - 2(1)(6)}}{2(1)}$ m1

$m = 2$, $m = 3.$ m1

$5^t = 2.$

$\ln 5^t = \ln 2$

$t = \dfrac{\ln 2}{\ln 5}$

$t = 0.4307$ A1

$5^t = 3$

$\ln 5^t = \ln 3$

$t = \dfrac{\ln 3}{\ln 5}$

$= 0.6826.$ A1

05

4. $P(x,y)$. $A(2,-3)$, $B(3,4)$

$AP : PB = 1:2.$

$\dfrac{\overline{AP}}{\overline{PB}} = \dfrac{1}{2}$ m1

$2\overline{AP} = \overline{PB}$

$2\sqrt{(x-2)^2 + (y+3)^2} = \sqrt{(x-3)^2 + (y-4)^2}$ m1

$4(x^2 - 4x + 4 + y^2 + 6y + 3) = x^2 - 6x + 9 + y^2 - 8y + 16$

$4(x^2 + y^2 - 4x + 6y + 7) = x^2 + y^2 - 6x - 8y + 25$

$3x^2 + 3y^2 \cancel{+} 10x + 32y + 3 = 0.$

$x^2 + y^2 - \dfrac{10}{3}x + \dfrac{32}{3}y + 1 = 0$, A1 i's a circle.

$c\left(\frac{5}{3}, -\frac{16}{3}\right)$ A1

05

$r = \sqrt{\left(\frac{5}{3}\right)^2 + \left(-\frac{16}{3}\right)^2 - 1}$

$r = 5.4975$ units A1

5.

$$y = \sqrt{4 + 3\sin x}$$

$$y^2 = 4 + 3\sin x$$

$$2y\frac{dy}{dx} = 3\cos x \quad m_1$$

$$2y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right) \cdot 2\frac{dy}{dx} = -3\sin x \quad m_1$$

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 3\sin x = 0$$

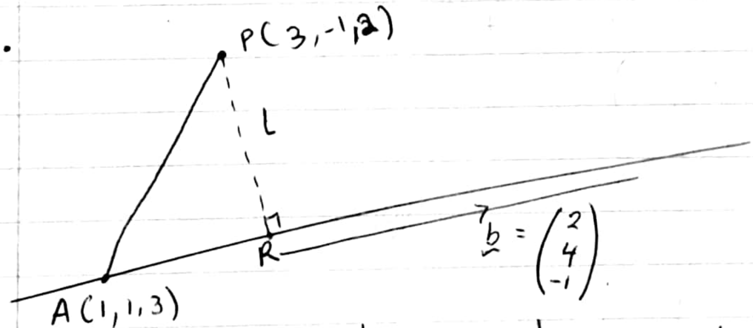
but $3\sin x = y^2 - 4 \cdot m_1$

05

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 - 4 = 0 \quad m_1$$

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4 \quad A_1 \quad \#$$

6.



$$l = \left|\frac{\overrightarrow{AP} \times b}{b}\right|$$

$$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$$

$$= \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \quad m_1$$

$$\overrightarrow{AP} \times b = \begin{vmatrix} i & -j & k \\ 2 & -2 & -1 \\ 2 & 4 & -1 \end{vmatrix} \quad m_1$$

$$= i\begin{vmatrix} -2 & -1 \\ 4 & -1 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 2 & -2 \\ 2 & 4 \end{vmatrix}$$

$$= 6i - 0j + 12k \cdot m_1$$

$$l = \frac{|6i + 12k|}{|2i + 4j - k|} \quad m_1$$

05

$$= \frac{\sqrt{(6)^2 + (12)^2}}{\sqrt{(2)^2 + (4)^2 + (-1)^2}}$$

$$= 2.9277 \text{ units} \quad A_1$$

OR.

6. $L = |\overrightarrow{PR}|$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$$

$$= \begin{pmatrix} 1+2\mu \\ 1+4\mu \\ 3-\mu \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2+2\mu \\ 2+4\mu \\ 1-\mu \end{pmatrix}$$

$\overrightarrow{PR} \perp \underset{\sim}{b}$.

$$\begin{pmatrix} -2+2\mu \\ 2+4\mu \\ 1-\mu \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = 0$$

$$-4+4\mu+8+16\mu-1+\mu=0$$

$$21\mu+3=0$$

$$\mu = -\frac{3}{21} = -\frac{1}{7}$$ A

$$\overrightarrow{PR} = \begin{pmatrix} -2+2(-1/7) \\ 2+4(-1/7) \\ 1-(-1/7) \end{pmatrix}$$

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$$\overrightarrow{PR} = \begin{pmatrix} -16/7 \\ 10/7 \\ 8/7 \end{pmatrix}$$

$$|\overrightarrow{PR}| = \sqrt{(-16/7)^2 + (10/7)^2 + (8/7)^2}$$

$$= 2.9277 \text{ units}$$ A

7. $x^2+kx-6k=0$, $x^2-2x-k=0$

Let the root be α

$$-\left|\begin{array}{l}\alpha^2+k\alpha-6k=0\\ \alpha^2-2\alpha-k=0\end{array}\right.$$

$$k\alpha+2\alpha-5k=0$$

$$\alpha=\frac{5k}{k+2} \quad m_1$$

$$\left(\frac{5k}{k+2}\right)^2-2\left(\frac{5k}{k+2}\right)-k=0 \quad m_1$$

$$25k^2-10k(k+2)-k(k+2)^2=0$$

$$k(25k-10k-20-k^2-4k-4)=0$$

$$k(-k^2-11k-24)=0$$

$$k(k^2+11k+24)=0 \quad m_1$$ a student can use the quadratic formulae.

$$k(k^2+3k+8k+24)=0$$

$$k(k(k+3)+8(k+3))=0$$

$$k(k+3)(k+8)=0 \quad m_1$$

$k=0$, $k=-3$, $k=-8$.

05

$\therefore$ $k=-3$, $k=-8$. A_1 (for both values of k).

8. $\frac{dv}{dt}=200cm^3s^{-1}$

$r=80mm$

$=8cm$.

$\frac{dA}{dt}=?$

$A=4\pi r^2$

$\frac{dA}{dr}=8\pi r \quad m_1$

$\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$

but.

$\frac{dr}{dt}=\frac{dr}{dv}\times\frac{dv}{dt}$

but $V=\frac{4}{3}\pi r^3$

$\frac{dv}{dr}=4\pi r^2 \quad m_1$

$\frac{dr}{dt}=\frac{1}{4\pi r^2}\times 200$

$=\frac{50}{\pi r^2} \quad m_1$

05

$\frac{dA}{dt}=8\cancel{\pi}\cancel{r}\times\frac{50}{\cancel{\pi}r^{\cancel{2}}} \quad m_1$

$=8\times\frac{5}{8}$

$=5\ cm^2s^{-1} \quad A_1$

SECTION B.

9. a,

$$\int \frac{1}{e^{2x}-1}\,dx.$$

let $u = e^{2x}-1$

$$\frac{du}{dx} = 2e^{2x} \quad m_1$$

$$dx = \frac{du}{2(u+1)} \quad m_1$$

$$= \int \frac{1}{2(u+1)u}\,du$$

but $\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{u+1}$ m_1

$1 \equiv A(u+1) + Bu.$

when $u=-1$ when $u=0$ m_1

$1 = -B$ m_1 $1 = A.$

$B = -1$

$$= \frac{1}{2}\int \frac{1}{u} + \frac{-1}{u+1}\,du \quad m_1$$

(08)

$$= \frac{1}{2}\ln u - \frac{1}{2}\ln u+1 + c$$

$$= \frac{1}{2}\ln(e^{2x}-1) - \frac{1}{2}\ln e^{2x} + c \quad m_1$$

$$= \frac{1}{2}\ln(e^{2x}-1) - x + c. \quad A_1$$

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b, $\displaystyle\int_0^{\pi/2} \frac{1}{1+\cos t}\,dt.$

$$= \int_0^{\pi/2} \frac{1}{1+2\cos^2 t/2 - 1}\,dt. \quad m1$$

$$= \int_0^{\pi/2} \frac{1}{2\cos^2 t/2}\,dt. \quad m1$$

$$= \frac{1}{2}\int_0^{\pi/2} \sec^2 t/2 \; dt.$$

$$= \Big[\tan t/2\Big]_0^{\pi/2}. \quad m1$$

04

$$= \tan \pi/4 - \tan 0/2.$$

$$= \underline{\underline{1}} \quad A1$$

10. a, $\left(\frac{1}{x^2}-x\right)^{18}$

let the co-efficient be A.

$$Ax^3 \equiv {}^{18}C_r\left(\tfrac{1}{x^2}\right)^{18-r}(-x)^r \quad m_1$$

$$= {}^{18}C_r\, x^{-36+2r}(-1)^r x^r$$

$$= {}^{18}C_r(-1)^r x^{-36+3r} \quad m_1$$

$$\Rightarrow \quad A = {}^{18}C_r(-1)^r, \quad x^{-36+3r} = x^3.$$

$$-36+3r = 3. \quad m_1$$

$$3r = 39$$

$$r = 13.$$

$$\therefore \quad A = {}^{18}C_{13}(-1)^{13}. \quad m_1$$

DS

$$= \underline{-8568} \quad A_1$$

b, $\sqrt{\frac{1+x}{1-x}} = (1+x)^{1/2}(1-x)^{-1/2}.$

for $(1+x)^{1/2} = 1 + \frac{1}{2}x + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{x^2}{2!} + \cdots$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots \quad m_1$$

$$(1-x)^{-1/2} = 1 + -\tfrac{1}{2}(-x) + -\tfrac{1}{2}\left(-\tfrac{3}{2}\right)\frac{(-x)^2}{2!} + \cdots$$

$$= 1 + \frac{x}{2} + \frac{3x^2}{8} + \cdots \quad m_1$$

$$\sqrt{\frac{1+x}{1-x}} = \left(1+\frac{x}{2}-\frac{x^2}{8}\right)\left(1+\frac{x}{2}+\frac{3}{8}x^2\right) \quad m_1$$

$$= 1 + \frac{x}{2} + \frac{3}{8}x^2 + \frac{x}{2} + \frac{x^2}{4} - \frac{x^2}{8} + \cdots$$

$$= 1 + x + \frac{1}{2}x^2 + \cdots \quad A_1 \quad \#$$

$$\sqrt{\frac{1+1/7}{1-1/7}} \approx 1 + \frac{1}{7} + \frac{1}{2}\left(\frac{1}{7}\right)^2 \quad B_1$$

$$\sqrt{\frac{8}{6}} \approx \frac{113}{98}$$

$$\sqrt{\frac{4}{3}} \approx \frac{113}{98} \quad B_1$$

$$\frac{2}{\sqrt{3}} \approx \frac{113}{98}$$

$$\sqrt{3} \approx \frac{196}{113} \quad B_1 \ \#$$

07

11. a) $2A + B = 45^\circ$

$$\tan(2A+B) = \tan 45^\circ \quad m_1$$

$$\frac{\tan 2A + \tan B}{1 - \tan 2A \tan B} = 1 \quad m_1$$

$$\tan 2A + \tan B = 1 - \tan 2A \tan B$$

$$\tan B + \tan 2A \tan B = 1 - \tan 2A$$

$$(\tan B)(1 + \tan 2A) = 1 - \tan 2A \quad m_1$$

$$\tan B = \frac{1 - \tan 2A}{1 + \tan 2A}$$

$$= \frac{1 - \frac{2\tan A}{1-\tan^2 A}}{1 + \frac{2\tan A}{1-\tan^2 A}} \quad m_1$$

06

$$= \frac{1 - \tan^2 A - 2\tan A}{1-\tan^2 A} \div \frac{1 - \tan^2 A + 2\tan A}{1-\tan^2 A} \quad m_1$$

$$= \frac{1 - 2\tan A - \tan^2 A}{1 + 2\tan A - \tan^2 A} \quad A_1 \ \#$$

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12 a)

b) $\tan^{-1}2x + \tan^{-1}3x = \pi/4$.

let $\tan^{-1}2x = A$ $\qquad$ $\tan^{-1}3x = B$

$\tan A = 2x$ $\qquad$ $\tan B = 3x$ m1

$A + B = \pi/4$.

$\tan(A+B) = \tan \pi/4$. m1

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\frac{2x + 3x}{1-(2x)(3x)} = 1 \text{ m1}$$

$5x = 1 - 6x^2$

$6x^2 + 5x - 1 = 0$.

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(6)(-1)}}{2(6)} \text{ m1}$$

06

$x = -1$ A1, $x = 1/6$ A1

12 a,

$\tan 61^\circ$

let $y = \tan x$.

$x = 60^\circ$, $\delta x = 1^\circ \times \frac{\pi}{180^\circ}$ M1

$= \frac{\pi}{180}$ rads.

$y = \tan 60^\circ$

$= \sqrt{3}$.

$\delta y \approx \left(\frac{dy}{dx}\right) \cdot \delta x$.

$\delta y \approx (\sec^2 x) \cdot \delta x$ M1

$= \frac{1}{\cos^2 60^\circ} \cdot \frac{\pi}{180}$

$= 0.0698$. M1

05

$\therefore \tan 61^\circ \approx \tan 60^\circ + \delta y$ M1

$= \sqrt{3} + 0.0698$

$= 1.802$.

$\approx \underline{\underline{1.80}}$ A1

b let $y = \operatorname{cosec} x$.

$y + \delta y = \operatorname{cosec}(x + \delta x)$ M1

$\delta y = \operatorname{cosec}(x + \delta x) - \operatorname{cosec} x$

$\delta y = \frac{1}{\sin(x + \delta x)} - \frac{1}{\sin x}$

$\delta y = \frac{\sin x - \sin(x + \delta x)}{\sin(x + \delta x)\sin x}$ M1

$\delta y = \frac{2\cos\left(\frac{x + x + \delta x}{2}\right)\sin\left(\frac{x - (x + \delta x)}{2}\right)}{\sin(x + \delta x)\sin x}$ M1

$\delta y = \frac{2\cos\left(\frac{2x + \delta x}{2}\right)\sin\left(-\frac{\delta x}{2}\right)}{\sin(x + \delta x)\sin x}$ M1

but for δx ~~very~~ very small, $\sin\left(-\frac{\delta x}{2}\right) \approx -\frac{\delta x}{2}$.

13 a,

$$\delta y = \frac{\cancel{2}\cos\left(\frac{2x+\delta x}{2}\right) \cdot \frac{-\delta x}{\cancel{2}}}{\sin(x+\delta x)\sin x}$$

$$\frac{\delta y}{\delta x} = -\frac{\cos\left(\frac{2x+\delta x}{2}\right)}{\sin(x+\delta x)\sin x} \quad m1$$

$$\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) \longrightarrow \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{\cos\left(\frac{2x}{x}\right)}{\sin x \sin x} \quad m1$$

$$= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\cot x \operatorname{cosec} x.$$

07

$$\therefore \frac{d \operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x. \quad \#$$

A1

13 a,

$x^2+4x-8y-4=0$.

i, $x^2+4x=8y+4$ m1

Completing squares.

$x^2+4x+(2)^2-(2)^2=8y+4$

$(x+2)^2=8y+4+4$ m1

$(x+2)^2=8(y+1)$: A1

In the form $x^2=4ay$.

03

ii, The cordinates are $(-2,-1)$. A1

01

b, $y=mx+c$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$b^2x^2+a^2y^2=a^2b^2$

$b^2x^2+a^2(mx+c)^2=a^2b^2$ m1

$b^2x^2+a^2(m^2x^2+2mcx+c^2)=a^2b^2$

$b^2x^2+a^2m^2x^2+2mca^2x+a^2c^2-a^2b^2=0$. m1

$(b^2+a^2m^2)x^2+(2mca^2)x+(a^2c^2-a^2b^2)=0$

At the point of touching, $b^2-4ac=0$.

$(2mca^2)^2-4(b^2+a^2m^2)(a^2c^2-a^2b^2)=0$. m1

$4m^2c^2a^4-4a^2(b^2+a^2m^2)(c^2-b^2)=0$

$m^2c^2a^2-(b^2+a^2m^2)(c^2-b^2)=0$

$m^2c^2a^2-(b^2c^2-b^4+a^2m^2c^2-a^2b^2m^2)=0$

$\cancel{m^2c^2a^2}-b^2c^2+b^4-\cancel{a^2m^2c^2}+a^2b^2m^2=0$

$-b^2c^2+b^4+a^2b^2m^2=0$ m1

$-c^2+b^2+a^2m^2=0$.

$c^2=b^2+a^2m^2$

$c^2=a^2m^2+b^2$ A1 #

$\frac{x^2}{9}+\frac{y^2}{4}=1$ and $y=x+1$

$m=1$, ~~c=1~~, $a=3$, $b=2$.

$c^2=3^2(1)^2+2^2$ m1

$c^2=13$ $c=\pm\sqrt{13}$.

$\therefore\ y = x + \sqrt{3}$ A1 and $y = x - \sqrt{13}$ A1 (08)

14. $z = x + yi$

$Arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{4}$

$Arg\left(\frac{x+yi-3}{x+yi-2i}\right) = \frac{\pi}{4}$ M1

$Arg\left(\frac{(x-3)+yi}{x+(y-2)i}\right) = \frac{\pi}{4}$

$Arg\left((x-3)+yi\right) - Arg\left(x+(y-2)i\right) = \frac{\pi}{4}$ M1

$\tan^{-1}\left(\frac{y}{x-3}\right) - \tan^{-1}\left(\frac{y-2}{x}\right) = \frac{\pi}{4}$ M1

let $\tan^{-1}\left(\frac{y}{x-3}\right) = A$, $\tan^{-1}\left(\frac{y-2}{x}\right) = B$

$\tan A = \frac{y}{x-3}$ M1 $\tan B = \frac{y-2}{x}$

$A - B = \frac{\pi}{4}$

(12)

$\tan(A-B) = \tan\frac{\pi}{4}$ M1

$\frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\frac{y}{x-3} - \frac{y-2}{x} = 1 + \left(\frac{y}{x-3}\right)\left(\frac{y-2}{x}\right)$ M1

$\frac{yx - (y-2)(x-3)}{x(x-3)} = \frac{x(x-3) + y(y-2)}{x(x-3)}$ M1

$yx - (yx - 3y - 2x + 6) = x^2 - 3x + y^2 - 2y$ M1

$3y + 2x - 6 = x^2 + y^2 - 3x - 2y$

$x^2 + y^2 - 5x - 5y + 6 = 0$ A1 Is a circle A1

$C\left(\frac{5}{2}, \frac{5}{2}\right)$ A1

$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 - 6}$

$r = 2.5495$ units A1

15. a,

$$\underset{\sim}{r} = \begin{pmatrix}1\\0\\-1\end{pmatrix} + \mu\begin{pmatrix}6\\-2\\1\end{pmatrix} + \beta\begin{pmatrix}-1\\3\\-7\end{pmatrix}$$

$$\underset{\sim}{n} = \begin{vmatrix} i & -j & k \\ 6 & -2 & 1 \\ -1 & 3 & -7 \end{vmatrix} \; M1$$

$$= i\begin{vmatrix}-2 & 1\\3 & -7\end{vmatrix} - j\begin{vmatrix}6 & 1\\-1 & -7\end{vmatrix} + k\begin{vmatrix}6 & -2\\-1 & 3\end{vmatrix} \; M1$$

$$= 11i + 41j + 16k . \; A1$$

let $R(x, y, z)$, $B(1, 0, -1)$

$$\overrightarrow{BR} = \overrightarrow{OR} - \overrightarrow{OB}$$

$$= \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} \; M1$$

$$= \begin{pmatrix}x-1\\y\\z+1\end{pmatrix} \; M1 .$$

07

$$\begin{pmatrix}x-1\\y\\z+1\end{pmatrix} \cdot \begin{pmatrix}11\\41\\16\end{pmatrix} = 0 \; M1$$

$$11x - 11 + 41y + 16z + 16 = 0$$

$$11x + 41y + 16z + 5 = 0 \; A1$$

9)

OR alternatively:

15a $r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} x-1 \\ y \\ z+1 \end{pmatrix} = \begin{pmatrix} 6\mu + \beta \\ -2\mu + 3\beta \\ \mu - 7\beta \end{pmatrix} \quad m1$$

from the i- component.

$\beta = 6\mu - x + 1$ m1

for j - component.

$y = -2\mu + 3(6\mu - x + 1)$

$y = -2\mu + 18\mu - 3x + 3$

$y = 16\mu - 3x + 3$

$\mu = \dfrac{y + 3x - 3}{16}$ m1

for k- component.

$z + 1 = \mu - 7(6\mu - x + 1)$ m1

$z + 1 = \mu - 42\mu + 7x - 7$ 07

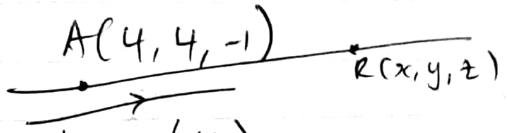
$z + 1 - 7x + 7 = -41\mu$

$z + 1 - 7x + 7 = -41\left(\dfrac{y + 3x - 3}{16}\right)$ m1

$16z + 16 - 112x + 112 = -41y - 123x + 123$ m1

$16z + 11x + 41y + 5 = 0$ A1

b,



$$\underset{\sim}{b} = \begin{pmatrix} 16 \\ 11 \\ 41 \end{pmatrix}$$

let $R(x, y, z)$

$\overrightarrow{AR} = \lambda \underset{\sim}{b}$. M1

$\overrightarrow{OR} - \overrightarrow{OA} = \lambda \underset{\sim}{b}$

$\overrightarrow{OR} = \overrightarrow{OA} + \lambda \underset{\sim}{b}$. M1

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 16 \\ 11 \\ 41 \end{pmatrix} \quad A1$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 + 16\lambda \\ 4 + 11\lambda \\ -1 + 41\lambda \end{pmatrix}$$

M1

$$\lambda = \frac{x-4}{16}, \quad \lambda = \frac{y-4}{11}, \quad \lambda = \frac{z+1}{41}$$

DS

$$\therefore \quad \frac{x-4}{16} = \frac{y-4}{11} = \frac{z+1}{41} \quad A1$$

16.

~~let the temperature of surrounding be~~
let the temperature of the liquid be θ

a,

$$-\frac{d\theta}{dt} \propto (\theta - 15) \cdot M1$$

$$\frac{d\theta}{dt} = -k(\theta - 15).$$

$$\frac{d\theta}{\theta - 15} = -k\,dt \cdot M1$$

$$\int \frac{d\theta}{\theta - 15} = -\int k\,dt \cdot M1$$

$$\ln(\theta - 15) = -kt + A$$

when $t = 0$, $\theta = 50°C$ M1

$$\ln(50 - 15) = -k(0) + A$$

$$A = \ln 35 \quad A1$$

$$\ln(\theta - 15) = -kt + \ln 35 \quad M1$$

at $t = 20\text{mins}$, $\theta = 35$

$$\ln(35 - 15) = 20k + \ln 35 \quad M1$$

$$\ln 20 - \ln 35 = -20k.$$

$$\ln(4/7) = -20k.$$

$$k = -\frac{1}{20}\ln(4/7). \quad A1$$

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$$\therefore \frac{d\theta}{dt} = \frac{1}{20}\ln(4/7)\,(\theta - 15). \quad A1$$

$$\ln(\theta - 15) = \frac{1}{20}t\,\ln(4/7) + \ln 35$$

at $t = 26.$ M1

$$\ln(\theta - 15) = \frac{26}{20}\ln(4/7) + \ln 35 \quad M1$$

$$\ln(\theta - 15) = 2.8278.$$

$$\theta - 15 = 16.9082.$$

$$\theta = 31.9082 \quad A1$$

$\therefore$ The temperature will be $31.9°$